A Streamline-Based Reservoir Simulation of the House Mountain Waterflood

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Abstract

Recent developments in streamline methods now allow for streamline-based reservoir simulators to be applied to a more general set of problems, previously could only be solved using conventional finite-difference methods. The streamline method we use is fully three-dimensional, can account for multiphase gravity effects, changing well rates, infill drilling, and non-uniform saturation conditions that result from infill drilling and/or water/oil contacts.

The purpose of this paper is to illustrate the application of the streamline method to a real field data set – in this case the House Mountain waterflood in central Alberta, Canada. The streamline simulation accurately models overall field historical data and on a well-by-well basis accurately models performance in 60% of the wells. Comparison of the streamline results to finite-difference results shows that both methods predict field and individual well performance similarly. The streamline simulation is only marginally faster than the conventional simulation for the coarse grid model due to the need to honor historical rate data. However, we illustrate the power of the streamline-based approach by simulating a downscaled 201,000 gridblock version of the original waterflood model in 10.5 hours on a standard workstation.

Introduction

The use of streamlines and streamtubes to model convective displacements in heterogeneous media has been presented many times since the seminal work by Muskat,[24–26] Fay and Prats,[13] and Higgins and Leighton.[16–18] Important subsequent contributions are due to Parsons,[27] Martin and Wegner,[22] Bommer and Schechter,[6] Lake *et al.*,[20] Emanuel *et al.*,[10] and Hewett and Behrens.[14]

Recently, streamline methods have resurfaced as a viable alternative to traditional finite-difference methods for large, heterogeneous, multiwell, multiphase simulations.[1,2,5,7,8,15,28,29,31–33] The success of our approach to using streamlines for reservoir simulation hinges on three important extensions to past streamline/streamtube methods: (1) the application to true 3D systems;[31] (2) accounting for changing well conditions such as changing rates, infill drilling, or producer-injector conversions[1,2] and (3) including gravity in multiphase flow.[2,7] With these extensions, we have been able to circumvent many of the limitations of current finite-difference and finite-element codes and simulate fluid flow in detailed, heterogeneous models using a fraction of current computational hardware and time.

We emphasize that reservoir simulation using streamlines is not a minor modification of current finite-difference approaches, but is a radical shift in methodology. The fundamental difference is in how fluid transport is modeled. In finite-differences, fluid movement is between explicit gridblocks, whereas in the streamline method, fluids are moved along a streamline grid that may be dynamically changing at each time step, and is decoupled from the underlying grid on which the pressure solution is obtained. By decoupling transport from the underlying grid we have noted large speed-up factors, minimization of numerical diffusion, and reduced grid orientation effects.[1–3,5,31–33] The inherent simplicity of the approach offers unique opportunities for integration with modern reservoir characterization methods. Examples have included ranking of equiprobable realizations, estimation of the uncertainty in production forecasts due to the uncertainty in reservoir properties, and rapid assessment of production strategies such as infill drilling and gas injection.[1,2,9,34]

While our previous examples have used synthetic data sets, the intent of this paper is to illustrate the performance of the streamline method with field data. Field examples with more limited streamline-based models have already been published. A two-dimensional field example was shown by Tyrie & Gimse[36] using a front tracking method along streamlines. Many field examples have also been published illustrating the use of a hybrid streamtube model that combined 2D areal and 2D cross-sectional models to approximate the 3D problem.[4,10,11,23] Recently Peddibhotla *et al.*[28] presented the application of a simplified 3D streamline model to a field example.

Summary of the Streamline Method

This section briefly summarizes our streamline method. For a detailed description we refer the reader to previous publications.[1,2,31]

Governing IMPES Equations The streamline method is an IMPES method. Ignoring capillary and dispersion effects, the governing equation in terms of pressure P for incompressible multiphase flow in porous media is given by

$$\nabla \cdot \vec{\vec{K}} \cdot (\lambda_t \nabla P + \lambda_g \nabla D) = 0, \qquad \qquad (1)$$

where the total mobility (λ_t) and the total gravity mobility (λ_g) are defined as

$$\lambda_t = \sum_{j=1}^{n_p} \frac{k_{rj}}{\mu_j}, \qquad \lambda_g = \sum_{j=1}^{n_p} \frac{k_{rj}\rho_j g}{\mu_j}. \qquad \qquad (2)$$

D represents a depth below datum. To determine the flow of the individual phases we also require a material balance equation for each phase j,[21]

$$\phi \frac{\partial S_j}{\partial t} + \vec{u}_t \cdot \nabla f_j + \nabla \cdot \vec{G}_j = 0. \qquad \qquad (3)$$

The total velocity $\vec{u}_t$ is derived from the 3D solution to the pressure field and application of Darcy's Law. The phase fractional flow term is given by

$$f_j = \frac{k_{ri}/\mu_j}{\sum_{i=1}^{n_p} k_{ri}/\mu_i}, \qquad \qquad (4)$$

and the gravity fractional flow term is given by

$$\vec{G}_j = \vec{\vec{K}} \cdot g \nabla D f_j \sum_{i=1}^{n_p} k_{ri}/\mu_i(\rho_i - \rho_j). \qquad \text{(5)}$$

In a conventional IMPES finite-difference simulator Eq. 3 is solved in its full three-dimensional form. With the streamline method, we decouple the 3D equation into multiple 1D equations that are solved along streamlines. For large problems, solving multiple 1D equations is much faster and more accurate than solving the full 3D problem, as we have already shown.[1,2,31]

Coordinate Transform Streamlines are launched from gridblock faces containing injectors. As the streamlines are traced from injectors to producers, we determine the time-of-flight[8,19] along the streamline, which is defined as

$$\tau = \int_0^s \frac{\phi}{u_t(\zeta)} d\zeta, \qquad \text{(6)}$$

and gives the time required to reach a point s on the streamline-based on the total velocity, $u_t(\zeta)$, along the streamline. To determine the coordinate transform, we rewrite Eq. 6 as

$$\frac{\partial \tau}{\partial s} = \frac{\phi}{|u_t|}, \qquad \text{(7)}$$

which can be rewritten as

$$|u_t| \frac{\partial}{\partial s} \equiv \vec{u}_t \cdot \nabla = \phi \frac{\partial}{\partial \tau}. \qquad \text{(8)}$$

Substituting Eq. 8 into Eq. 3 gives

$$\frac{\partial S_j}{\partial t} + \frac{\partial f_j}{\partial \tau} + \frac{1}{\phi} \nabla \cdot \vec{G}_j = 0. \qquad \text{(9)}$$

Eq. 9 is the governing pseudo-1D material balance equation for phase j transformed along a streamline coordinate. It is pseudo-1D since the gravity term is typically not aligned along the direction of a streamline.

To solve Eq. 9 we simply split the equation into two parts.[2,7] First a convective step along streamlines governed by

$$\frac{\partial S_j^c}{\partial t} + \frac{\partial f_j}{\partial \tau} = 0, \qquad \text{(10)}$$

which includes boundary conditions at the wells, is taken to construct an intermediate saturation distribution S_j^c. Then, a gravity step is taken along gravity lines and saturations are moved using

$$\frac{\partial S_j}{\partial t} + \frac{g}{\phi} \frac{\partial G_j}{\partial z} = 0, \qquad \text{(11)}$$

with S_j^c as the initial condition to construct S_j at the next time step. For simplicity we have assumed that the gravity lines are aligned in the z coordinate direction. Eq. 10 is solved numerically using single point upstream weighting scheme explicit in time. Eq. 11 is solved using an explicit upstream weighting method outlined by as Sammon.[30] An additional advantage of decoupling Eq. 9 in this way is that Eq. 11 is only solved in flow regions where gravity effects are important. For example, in locations where fluids are completely segregated, Eq. 11 will not be solved, since $\frac{\partial G}{\partial z}$=0.

Time Stepping To model field scale displacements our underlying assumption is that the streamline paths change with time due to the changing mobility field and/or changing boundary conditions. Thus the pressure field is updated periodically in accordance with these changes. By using numerical solutions along the recalculated streamline paths the method accounts for the non-uniform initial conditions now present along the recalculated paths.

To move the 3D solution forward in time from t^n to $t^{n+1} = t^n + \Delta t^{n+1}$ we use the following algorithm:

1. At the start of a new time step, t^{n+1}, solve for the pressure field P using Eq. 1 in the IMPES formulation. We solve Eq. 1 using a standard seven-point finite difference scheme, with no-flow boundary conditions over the surface of the domain and specified pressure or rate at the wells.

2. Apply Darcy's Law to determine the total velocity at gridblock faces.

3. Trace streamlines from injectors to producers. For each streamline we do the following: (a) While tracing a streamline, the current saturation information from each gridblock that the streamline passes through is remembered. In this manner, a profile of saturation versus τ is generated for the new streamline; (b) Move the saturations forward by Δt^{n+1} by solving Eq. 10 numerically in 1D. Map the new saturation profile back to the original streamline path.

4. Average all the streamline properties within each gridblock of the 3D domain to determine the saturation distribution at t^{n+1}.

5. If $G_j \neq 0$ include a gravity step that traces gravity lines from the top of the domain to the bottom of the domain along $\vec{g}$. For each gravity line we do the following: (a) While tracing a gravity line, the saturation distribution calculated in the convective step as a function of z is remembered; (b) The saturations are moved forward by Δt^{n+1} using Eq. 11. Map the new saturation profile back to the original gravity line.

6. If $G_j \neq 0$ average all gravity line properties within each gridblock of the 3D domain to determine the final saturation distribution at t^{n+1}.

7. Return to Step 1.

A key reason for the large speedup factors in the streamline method is the fact that Δt, the time step size for a convective step that includes a pressure solve, can be orders of magnitude larger than the time step size in conventional simulators. This is a result of eliminating the global grid CFL condition by decoupling fluid movement from the underlying grid. An important consideration in field simulations however, is that the time step size in the streamline method can be limited by the need to honor changing well conditions. Thus we expect speedup factors to be smaller for simulations that must honor historical production information since the pressure field is recomputed every time the well conditions change, as opposed to using the method in a forecast mode.

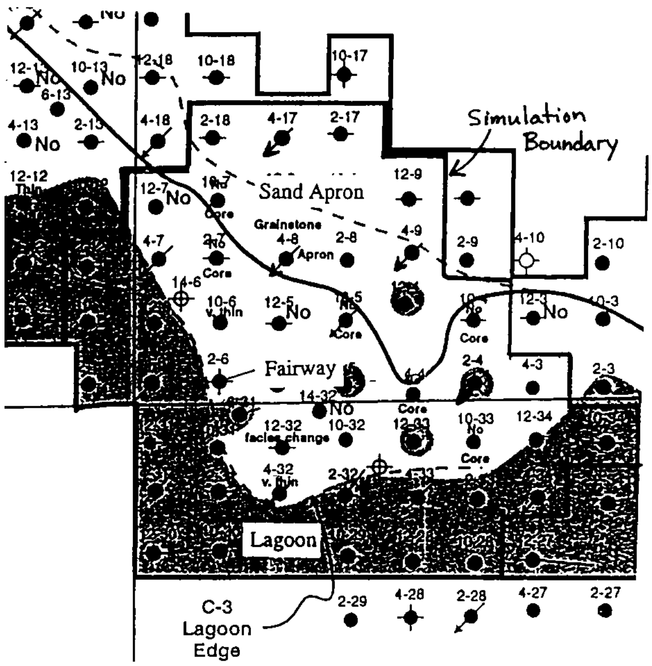

Figure 1: **House Mountain reservoir study area showing well locations and facies distributions.**[12]

House Mountain Waterflood

The House Mountain waterflood is located in central Alberta, Canada and was discovered in 1963. The field is operated by Shell Canada and currently consists of 279 wells. A waterflood was implemented in 1966 with reservoir pressures always maintained above the bubble point pressure.

Recently a 56 well portion of the reservoir has been simulated, using the Shell in-house conventional simulator MORES, to improve the understanding of the waterflood.[12] Our purpose is to model the same 56 well data set using the streamline method and compare the history match with both the conventional numerical model and the production data.

Conventional Simulation Model Description House Mountain is a carbonate reservoir consisting of three main cycles, each the result of a deepening event.[35] Within the simulation area there are three main areal subdivisions; Sand Apron, Fairway, and Lagoon (Fig. 1). In brief, the Sand Apron is poor quality and consists mainly of Cycle 2, the Fairway is the best quality reservoir and contains all three cycles, and finally the Lagoon is poor quality reservoir in all three cycles.

Fine scale 800,000 gridblock general corner-point permeability and porosity models were developed using Stratamodel. The models were based on core data from 80% of the wells in the study area. These models were upscaled to 70×70×7 and contained 21,000 active gridblocks. For this grid, permeability was log normally distributed and ranged from 0.08 mD to 800 mD with a coefficient of variance of 2, porosity values ranged from 0.2% to 12.5% after including net-to-gross factors. Additionally, transmissibility barriers in the Fairway and Lagoon portions of the reservoir were also included between simulation layers 3 & 4 and simulation layers 5 & 6 to account for the barriers between Cycles 3 & 2 and Cycles 2 & 1 respectively (Fig. 2).

The MORES simulation model also included three main rock types, the primary difference being the residual oil saturation of each type. For the three rock types, residual oil saturation ranged from 30% in the Fairway, to 40% in the Sand Apron, to 50% in the Lagoon. Corey type relative permeability curves were assumed with an end-point water relative permeability of k_{rw}=0.3 and an end-point oil relative permeability of k_{ro}=1.0 for all rock types. At reservoir conditions the oil properties are μ_o=0.55 cp and ρ_o= 815 kg/m^3, and the water properties are μ_w=0.38 cp and ρ_w=1005 kg/m^3. Based on a Buckley-Leverett displacement analysis, this system would be considered a favorable mobility ratio displacement. Since the displacement is stable, numerical diffusion effects are small in the conventional numerical simulation.

Production data for each well was provided at half-year intervals from 1966-1996. Aside from standard well shut-ins and start-ups, there were two producer-injector conversions, 14 wells were abandoned, 2 infill wells were added, and 2 horizontal redrill wells were added. To match reservoir pressures and account for fluid migration out of the simulation boundary in the MORES model, allocation factors were assigned to water injection volumes of fringe injectors and pseudo production wells were added to the western grid boundary.

Modifications to Simulation Model Before the model could be simulated using the streamline simulator 3DSL, the following assumptions were made.

1. The model was assumed to be incompressible. This is a satisfactory assumption since the waterflood was never allowed to drop below the bubble point pressure and the voidage replacement ratio (VRR) was approximately 1.0. A value of $B_o = 1.28$ was assumed to convert surface oil rates to reservoir rates and a value of $B_w = 1.0$ was assumed for the water phase rates.

2. To honor a VRR=1, the pseudo wells were set to constant bottom hole pressure constraint and allowed to produce or inject depending on the historical production and injection rates. Thus the western edge of the model represents a leaky boundary where fluids migrated back and forth.

3. The grid geometry was assumed to be a standard Cartesian grid rather that a general corner-point grid. This assumption only affected the vertical coordinate direction. Given that the reservoir has minimal dip and vertical relief changes are only on the order of 1-2 meters, the loss in vertical relief detail on the standard Cartesian grid is minimal.

4. Only a single set of relative permeability curves could be used within 3DSL. Thus only a single residual oil saturation was input to the model. An average value of S_{or}=0.4 was assumed.

5. The initial period before waterflooding (1963-1966) was not modeled, and the initial oil saturation was assumed to be uniform at the start of 1966.

To summarize, the assumptions required to run the House Mountain data set with 3DSL, having the largest impact on

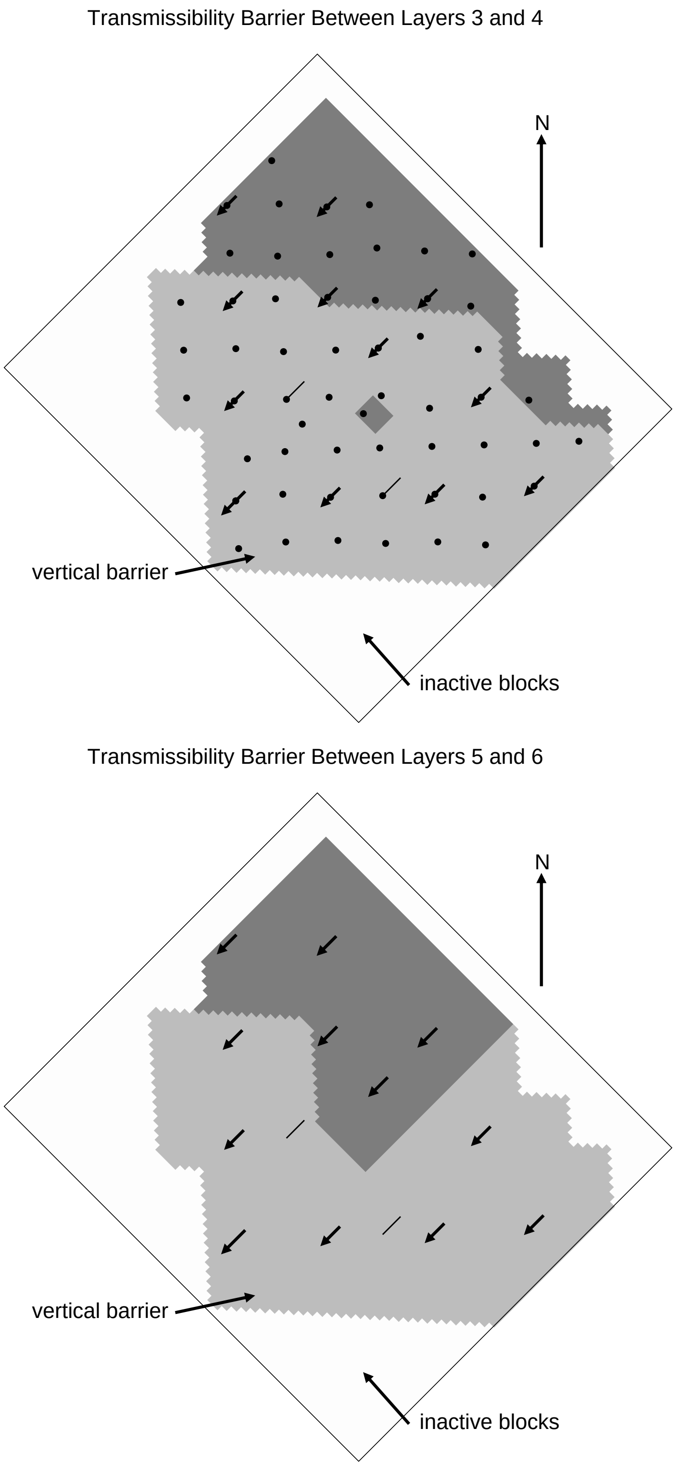

Figure 2: **Light gray areas indicate location of transmissibility barriers in the vertical direction between the layers noted.**

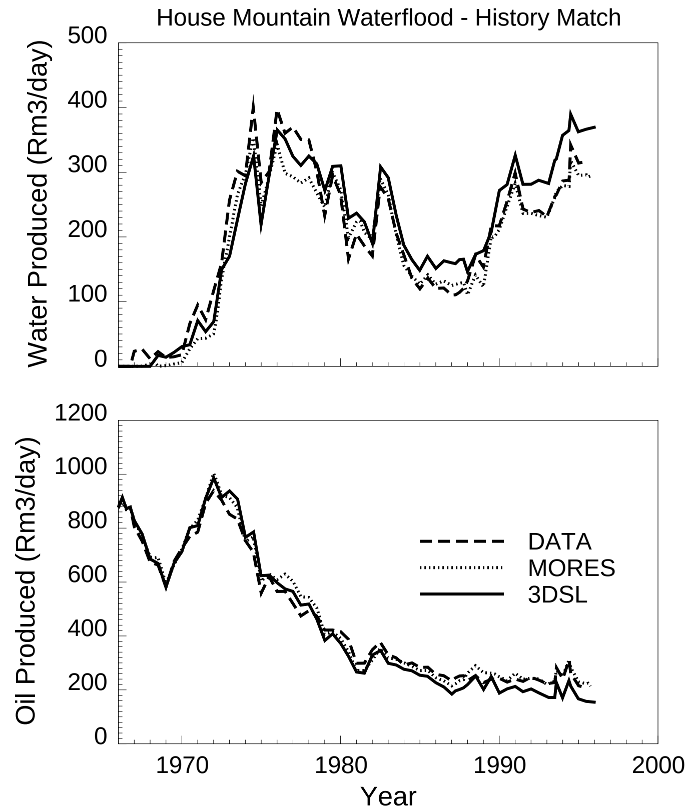

Figure 3: **Comparison of historical data with history matches for the streamline simulator (3DSL) and the conventional simulator (MORES) for the 70×70×7 grid model.**

the streamline simulation model results are the restriction to a VRR=1 and a single set of rock relative permeability curves.

History Match with Streamline Model For both simulation models, the well boundary conditions for the history matching period were the historical total liquid rate production at each producer and the historical total liquid injection rate at each injector. Thus the indicators of how good the history match is, are the individual phase production rates after water breakthrough. It is important to note that aside from the assumptions given above, the data was treated as is. No attempt was made to history match the streamline model results. However, it is fair to say that adjustment of the injection rate allocation factors could have been made to compensate for the lack of varying rock permeability curves.

Fig. 3 compares the historical field oil and water production rates to the streamline simulation and the conventional simulation. Pre 1985 there is excellent agreement between both numerical methods and the historical data on a field wide basis. Post 1985 the streamline simulation slightly over predicts water production primarily due to an over prediction in water rates at two southern Fairway producers (4-5 and 10-32 Fig. 4). This most likely a result of using only a single average set of relative permeability curves in the streamline simulation. The average residual oil saturation is 10% higher than the value assigned

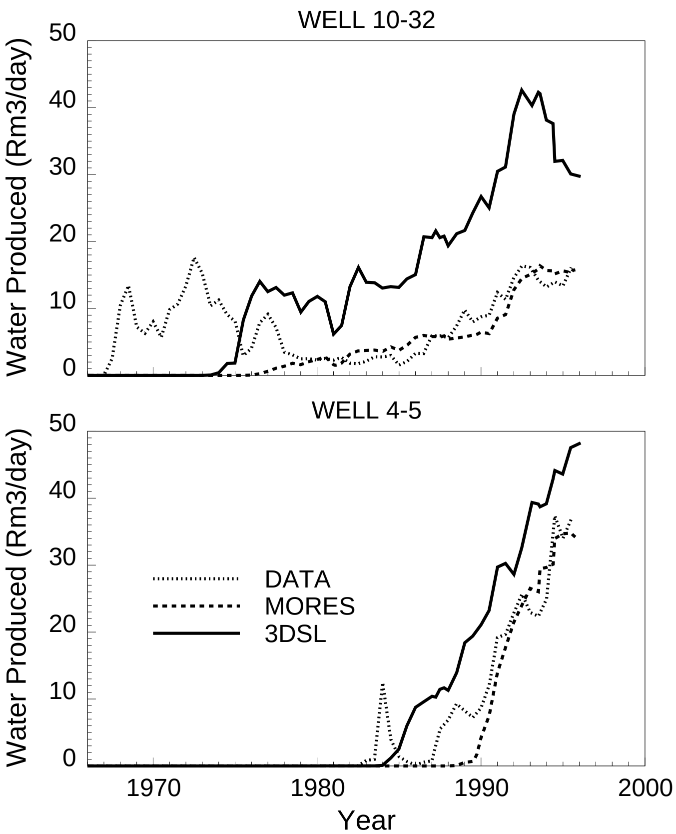

Figure 4: **Two Fairway producers with water rates over predicted in the streamline simulation model.**

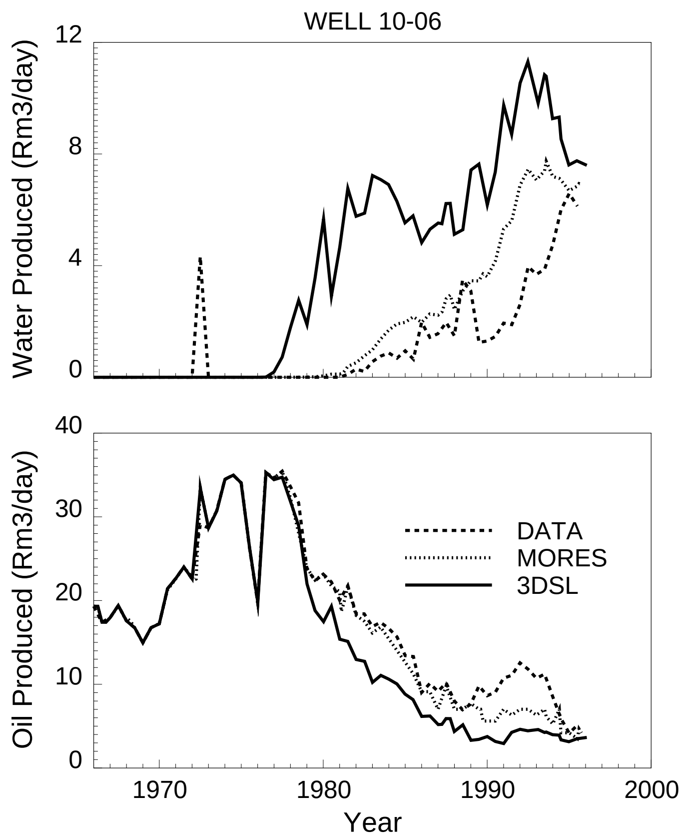

Figure 5: **Example of a poorer history match well for the streamline simulation than the conventional simulation.**

to the Fairway region in the MORES model. Thus where water breakthrough has occurred in the Fairway, water rates are over predicted compared to the MORES model.

On a well-by-well basis, out of the 45 producers, 8 were considered to have a poorer match with historical data for the streamline model than for the MORES model. An example of a poorer match is well 10-6 (Fig. 5) which again is a Fairway producer. However, 60% of the wells did have an excellent history match in the streamline model, compared with 76% for the original simulation model. Wells 12-8 (Fig. 6) and 4-4 (Fig. 7) are examples of excellent streamline model history matches with water breakthrough performance predicted better than the MORES model.

Model Run Time For this small 21,000 gridblock model the maximum time step size was basically dictated by the well events which occurred every half year. The streamline simulation required 65 time steps, coinciding with the 65 well events between 1966 and 1996, while MORES required 72 time steps over the same simulation period (fully implicit scheme). Thus the speedup factor of the streamline method over the fully implicit scheme for this coarse grid model was minimal. Note that an ECLIPSE IMPES model was run on the streamline model data set and required 730 time steps.

Downscaled Streamline Model For the small model the streamline method proved to be no faster than the conventional implicit method, because of the need to honor historical data. However, the advantage of the method is best illustrated by simulating a finer grid. Rather than perform a history matching study on a reduced upscaled version of the original 800,000 gridblock model, we simply downscaled the original $70\times70\times7$ grid to $210\times210\times7$ grid (201,000 active gridblocks). In the process of downscaling no new information was added to the permeability or porosity fields. However, the number of gridblocks between wells has increased by a factor of 3. Fig. 8 compares the history match of the fine scale model to the coarse scale model and the historical data. Both streamline models have similar profiles with minor differences due to the improved resolution of the displacement front between wells. Note that a conventional model could not be run due to computer constraints. The downscaled model was 15 times slower than the upscaled model and required 10.5 hours CPU time. The number of time step required did not change.

With the ability to easily run a fine grid model, one option is to incorporate more heterogeneity and revisit the history matching process. However, the history match is already quite good. A second choice is to keep the grid resolution between wells the same as in the coarse grid model but include more wells in the simulation. The finescale model could easily include all 279 wells. A full field simulation would also reduce the need to ac-

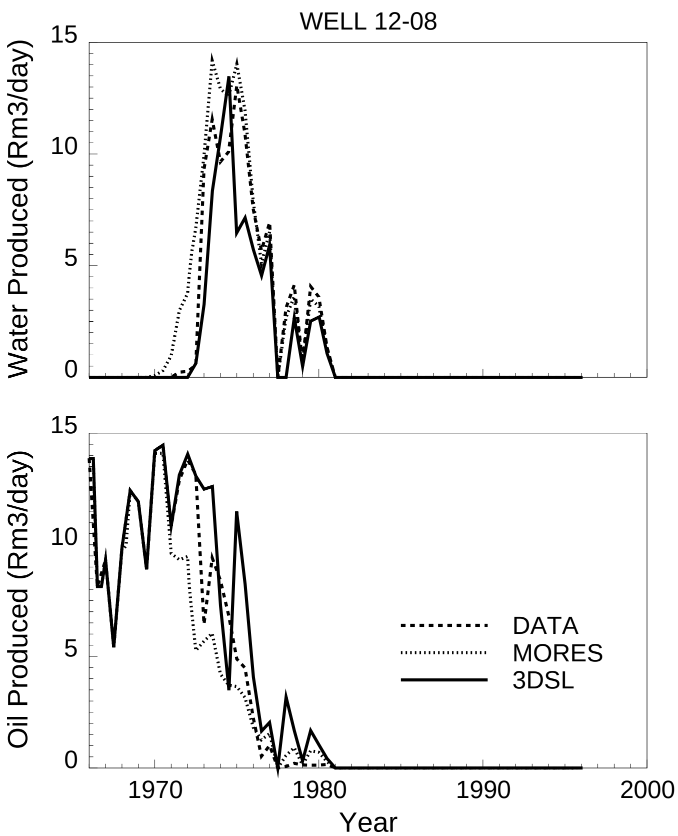

Figure 6: **History match for well 12-08.**

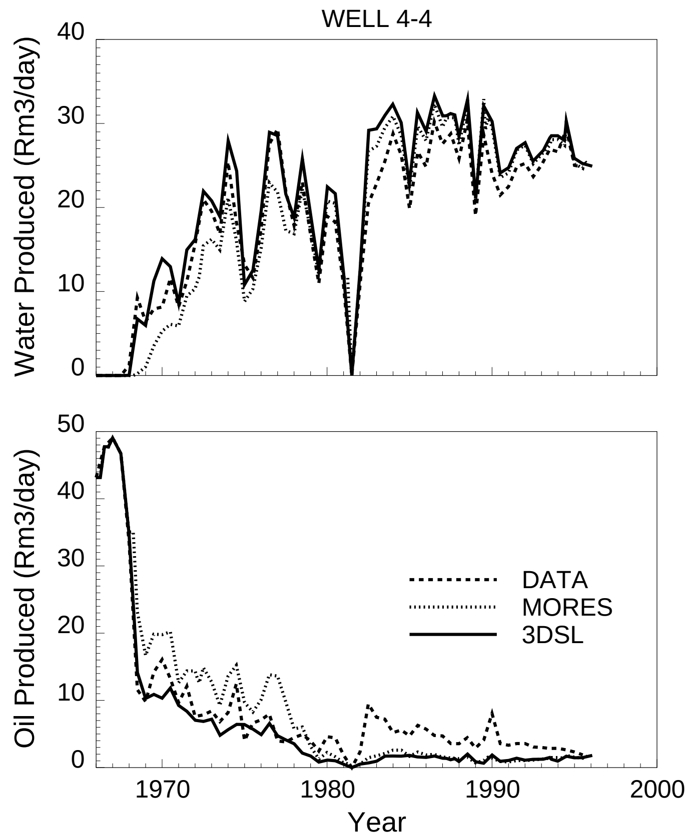

Figure 7: **History match for well 4-4.**

count for leak-off across simulation boundaries.

Discussion

This paper illustrates that the streamline method can accurately model true 3D field waterflood displacements. Although some modifications to the data set were required, these were only necessary because of current limitations in 3DSL. In a general streamline method, compressibility and rock dependent relative permeability curves can be modeled and are areas of current research. In principle corner-point geometries can also be included. However corner-point methods were introduced primarily due to the lack of fine gridding available and the desire to maintain flow paths in the direction of facies types. The streamline method allows for the use of finer grids. Furthermore, fluids are transported along the curvilinear streamline grid, rather than between discrete gridblocks and thus corner-point geometry grids are not required.

For the small model studied here, speedup factors were minimal for the streamline method due to the need to honor historical production data at half year intervals. To improve performance and reduce time spent on the history matching process one could choose to lump the 30 years of production data into larger time intervals of a year, for example. However, now that we have illustrated that the method can account for production data, a more interesting study is in how the method performs with larger field models or in forecast mode where boundary conditions change less frequently. We have already shown that when boundary conditions are relatively constant, the method can be up to 100 times faster than a conventional simulation for large multiwell problems.[1,2]

Conclusions

We have demonstrated that by mapping numerical solutions along streamlines and using operator splitting to account for multiphase gravity effects, the streamline method is applicable to true three- dimensional field scale waterflood simulations with only minor approximations to data input. Due to the historical production constraints, the method was only marginally faster than the fully implicit FD simulation. However, the method gave an excellent history match on a field basis, and predicted individual well responses almost as well as the conventional simulation. Fine tuning of the injection rate allocation factors would have improved the match of the streamline simulation. Additionally, we showed that the method is ideally suited to solving large gridblock models that cannot be solved using conventional methods. A 201,000 gridblock model with 30 years of production data was simulated on an average size workstation in 10.5 hours. Clearly the streamline method can either include larger simulation areas with more wells and/or reduce the need for substantial upscaling.

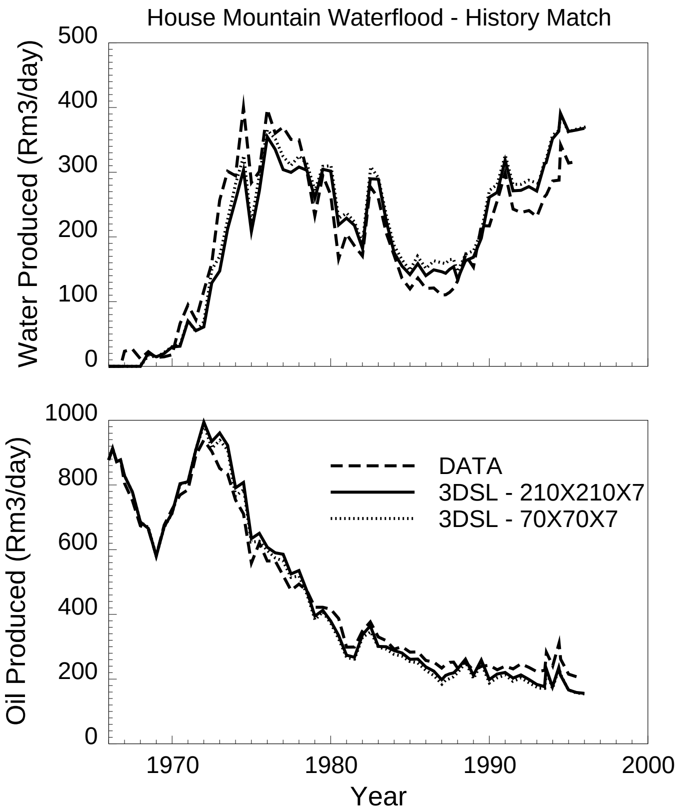

Figure 8: **Comparison of historical data with history matches for the streamline simulator on the 70×70×7 grid model and the downscaled 210×210×7 grid downscaled model.**

Acknowledgments

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Nomenclature

D	=	depth from datum, L
f_j	=	fractional flow of phase j, dimensionless
$\vec{G}_j$	=	gravity fractional flow term of phase j, L/T
g	=	gravitational acceleration constant, L/T^2
$\vec{\vec{K}}$	=	absolute permeability tensor, L^2
k_{rj}	=	relative permeability of phase j, dimensionless
n	=	number of pressure time steps
n_p	=	number of phases
P	=	Pressure M/T^{2}L
S_j	=	saturation of phase j, dimensionless
S_j^c	=	intermediate saturation profile after convective step
s	=	spatial distance coordinate along a streamline, L
t	=	time, T
$\vec{u}_t$	=	total Darcy velocity, L/T
VRR	=	voidage replacement ratio, dimensionless
z	=	vertical coordinate direction, L
Δt	=	time step size between pressure solves, T
ϕ	=	porosity, dimensionless
ρ_j	=	density of phase j, M/L^3
ζ	=	local streamline coordinate, L
λ_g	=	total gravity mobility, L/T
λ_t	=	total mobility, L^{3}T/M
τ	=	time of flight, T
μ_j	=	viscosity of phase j, M/TL

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